

Nonequilibrium Phase Transition in Ferromagnetic Model Systems

*Thesis Submitted for the partial fulfillment of the requirements for
the degree **Doctor of Philosophy in Science***

By

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To
My Parents and lovely family
&
My Supervisor

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It is now when I am at this moment to submit my thesis I turn around to see in wonder the journey I have traveled throughout the last five years. The dream that I cherished has come true. It has been a wonderful journey full of exciting and unforgettable memories. I have learnt a lot, though a little in the sea of knowledge. Like a ray of light in the dark, like a shadow under the heat of the sun, like blossoms full of wild fragrance, like a friend walking along with or like a rain-drop on a sapling calling to rise again, I have met with many people, over the years during my time as a Ph.D. student, who have been a lot to me in these days. A little joy and happiness is peeping around my senses in this moment and I acknowledge those persons from the deepest core of my heart.

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DECLARATION

I hereby declare that this thesis contains original research work carried out by me under the guidance of Mukdish Acharyya, Professor, Department of Physics, Presidency University, Kolkata, India as part of the PhD programme.

All information in this document have been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by these rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

I also declare that, this work has not been submitted for any degree either in part or in full to any other institute or University before.

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CERTIFICATE

This is to certify that the thesis entitled “Nonequilibrium Phase Transition in Ferromagnetic Model Systems” submitted by Shri Ajay Halder, who got his name registered for PhD programme under my supervision (Registration Number R-14RS09140064 and date of registration 12th August 2016) and that neither his thesis nor any part of the thesis has been submitted for any degree / diploma or any other academic award anywhere before.

Muktish Acharyya 2.9.2019
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Abstract:

In this thesis I have mainly discussed the dynamical responses and behaviours of a ferromagnetic system under different kind of magnetic field variations that are varied in time as well as in space, specially in the form of propagating magnetic wave and standing magnetic wave using Monte-Carlo simulation. These magnetic field waves cause the ferromagnet to undergo nonequilibrium phase transition; depending on the temperature, amplitude and wavelength of the magnetic field and the strength of anisotropy. In an Ising ferromagnet driven by standing magnetic field wave the high temperature phase is found to be quite different from that observed when propagating magnetic wave was passing through the Ising ferromagnet. Dynamical phases having similar attributes are formed in Blume-Capel ($S = 1$) ferromagnet when placed under these kind of waves. However, in BC ferromagnet the strength of anisotropy affects the transition temperature as well as the morphological structure of the ferromagnetic spins. The studies regarding general characteristics of spin- S Ising ferromagnet excited by magnetic field waves revealed that the nature of dynamic phase transition is similar to the earlier studies but the transition temperature is found to decrease towards a limiting value with increase in the value of S . The values of critical exponents of spin- $\frac{1}{2}$ Ising ferromagnet driven by propagating magnetic field wave are found to be very close to those obtained in Onsager's solution for equilibrium ferro-para phase transition. This indicates that the nonequilibrium phase transition in an Ising ferromagnet driven by propagating magnetic wave belongs to the same Universality Class of equilibrium Ising ferromagnet.

List of Publications:

1. **Standing magnetic wave on Ising ferromagnet: Nonequilibrium phase transition.**
Ajay Halder and Muktish Acharyya *J.Magn.Magn.Mater.* **420**(2016)290.
2. **Blume-Capel ferromagnet driven by propagating and standing magnetic field wave: Dynamical modes and nonequilibrium phase transition.**
Muktish Acharyya and Ajay Halder *J.Magn.Magn.Mater.* **426**(2017)53.
3. **Nonequilibrium Phase Transition in spin-S Ising ferromagnet driven by Propagating and Standing magnetic field wave.**
Ajay Halder and Muktish Acharyya *Commun. Theor. Phys.* **68**(2017)600.
4. **Universality class of the nonequilibrium phase transition in two dimensional Ising ferromagnet driven by propagating magnetic field wave.**
Ajay Halder and Muktish Acharyya *Applied Mathematics* 10 (2019) 568-577.

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