Chapter 4

Statistical fluctuation of attractive BEC in harmonic trap

For the present study, in an almost spherically symmetric harmonic oscillator trap of frequency 144.6 Hz, we considered the condensate of ^7Li atoms. The value of C_6 and r_c are chosen in such a way to have $a_s = -27.3$ Bohr with asymptotic short-range correlation function η_{ij} having zero node. The system parameters mimic the RICE University trap. With the proper choice of the system parameters, the effective potential $\omega_0(r)$ is calculated in which the whole condensate moves like a single quantum stuff. The ^7Li condensate is attractive interacting BEC. Due to the interaction between the atoms, the density of the condensate near the trap centre increases if we increase the particle number. Beyond a critical particle number present in the trap BEC collapses because the kinetic energy is unable to balance the interaction energy. The effective potential has two separate regimes -metastable region and narrow attractive well region. The metastable region of the effective potential is holding our condensate and if the number of atoms increases then the intermediate barrier will not be able to hold the system in the metastable region they will move to the narrow attractive well region to form a cluster. So, we will restrict ourselves in the metastable atom number limit $N \sim 1300$ where the critical particle number for the collapse is around 1450 [4, 38, 39]. The full energy spectrum of the BEC, moving in this metastable effective potential regime, is calculated. The spectrum corresponds to the solution of the temperature-independent coupled differential equation obtained from the full time-independent many-body

Schrödinger equation. The temperature dependence of different state properties of BEC comes through the partition function calculation. We have considered the canonical ensemble to calculate the partition function disregarding the grand canonical and microcanonical approach. Our main focus of this chapter is to present the result for the ground state probability distribution, condensate fraction, ground state atom number fluctuation and several orders of central moment for the attractive interacting BEC and how they are affected by the interaction. We will explore, the possibility of phase transition in this kind of mesoscopic system where we cannot reach truly thermodynamic limit due to the possibility of collapse. Due to this collapse scenario, the attractive interacting system is a very appealing one to study in detail. The stability of the attractive BEC has been extensively studied under several realistic situations [39, 41]. The earlier calculations with interacting bosons exhibit different static and thermodynamic properties but we didn't find any research work related to the realistic experimental situation. Some previous works with interacting Bose gas basically considered that the particles are trapped in a rectangular box with periodic boundary conditions [113, 145]. It is already known that, the condensate trapped in such a box-confinement has the zero momentum component of the atomic field which corresponds to the ideal gas. Also for the Bogoliubov quasi-particle approach, the quasi-particles are treated as independent bosons [43, 94, 107, 108, 113, 115, 123, 125, 140, 146]. So, none of these earlier works truly considered the real experimental situation considering the interacting bosons in a harmonic trap. We will consider real interacting Hamiltonian to push our many-body calculation beyond mean-field calculation.

4.1 Ground state probability distribution

The distribution of the ground state atoms is a very important thermodynamic quantity. How this distribution function is effected with increment in the net attractive interaction has been discussed. We have plotted the ground state probability distribution function P_{n_0} as a function of ground state occupation number n_0 in figure 4.1. The thermal evolution of P_{n_0} can be viewed from the same plot as it contains P_{n_0} for several temperature set - $t = 0.2T_c^0$, $= 0.5T_c^0$ and $= 0.8T_c^0$. T_c^0

is the temperature where a transition from condensed BEC to classical Bose gas takes place for ideal Bose gas. By the temperature dependence of several thermodynamic quantities, we can probe the process of phase transition in the trapped interacting Bose gas.

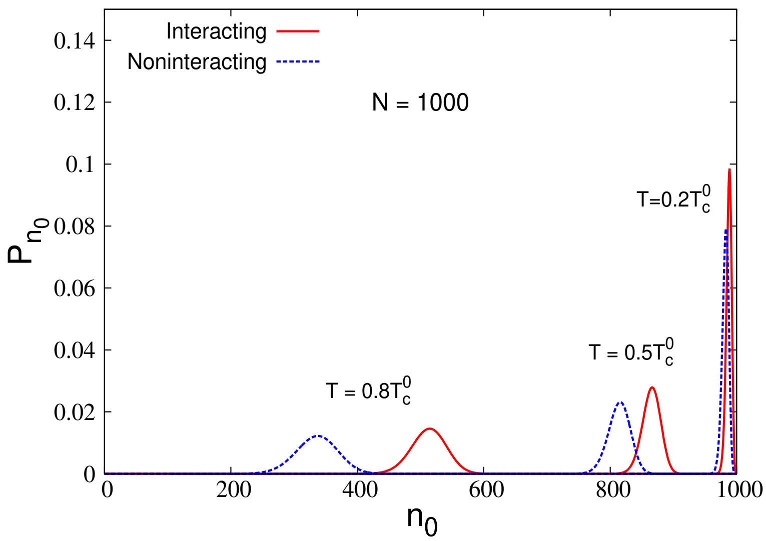

Figure 4.1: Ground state probability distribution (P_{n_0}) against n_0 for $N = 1000$ interacting and noninteracting bosons considering ^7Li atoms in the harmonic trap.

With increase in the temperature, we infer the depletion in the number of atoms in the ground state and enhancement in the number of atoms in the excited state. For very small temperature ($= 0.2T_c^0$), approaching close to transition temperature T_c^0, we see the highest peak centered at the large n_0 value which means more and more number of atoms are accumulating in the ground state. As the temperature increases, the depletion in the ground state becomes prominent showing the smaller peak and also the peaks are centered at smaller particle number region. The distribution functions for the ideal bosons are also presented for comparison for the same set of temperatures as reported for the interacting case. Exact analytical formulas for the distribution of uncondensed atoms of ideal Bose gas (IBG) had been suggested by several groups [124, 126, 107]. An earlier work with repulsive BEC by utilizing CPHEM depicted that, the obtained results are in very good agreement with the analytical results [43]. As the temperature increases, due to the interaction between the atoms, the population in the excited state enhances and depletion in the ground state is observed for $T > T_c^0$. But for $T << T_c^0$ we observe sharply peaked distribution centered at a large n_0 limit. However, the peak for a specific temperature is higher for the attractive interacting Bose gas in comparison with the ideal case. Hence, we can conclude that attractive interaction between the atoms enhances the pair correlation which increases the population in the ground state or in other words enhances the condensation process.

4.2 Condensate fraction

Now, we will discuss an important thermodynamic quantity - the condensate fraction $\frac{n_0}{N}$. n_0 is the average number of particles in the ground state and N is the total number of trapped atoms. In some earlier studies, the temperature dependence of the condensate fraction for repulsively interacting Bose gas in harmonic and also in anharmonic trap have already been studied [43, 42]. However, the temperature dependence of the same for attractive interacting BEC is still unknown. Here, we discuss the temperature dependence of the condensate fraction for the attractive ^7Li BEC by utilizing the ground state probability distribution function. In figure 4.2, we have represented the condensate fraction for several number of attractively interacting bosons, *viz.*, $N = 500$, 1000 and 1300. During the temperature evolution, the interaction strength is kept fixed. The condensate fraction for the IBG is also presented for comparison. We see that the change in the condensate fraction for the interacting case is more sudden near the critical temperature. The curves for the attractive interacting Bose gas shifted to the right side from the non-interacting ideal gas. So, attractive interaction between the Bose particles stimulates the BEC process and yields an enhancement in the ground state occupation number n_0 at any intermediate temperature as compared to the IBG. The reason for enhancement in the condensate fraction is related to the effect of the interaction energy between the particles. Bosons in different but mutually overlapping states interact more than the two particles interacting in a single energy state. Since, two bosons in the same state correspond to the lowest energy configuration so multiple occupations of a single particle energy state is more energetically favourable. Such a kind of effect is called an attraction in the momentum space [139, 116]. From the figure 4.2, one can draw the conclusion that attractive interaction supports the achievement of high-temperature BEC.

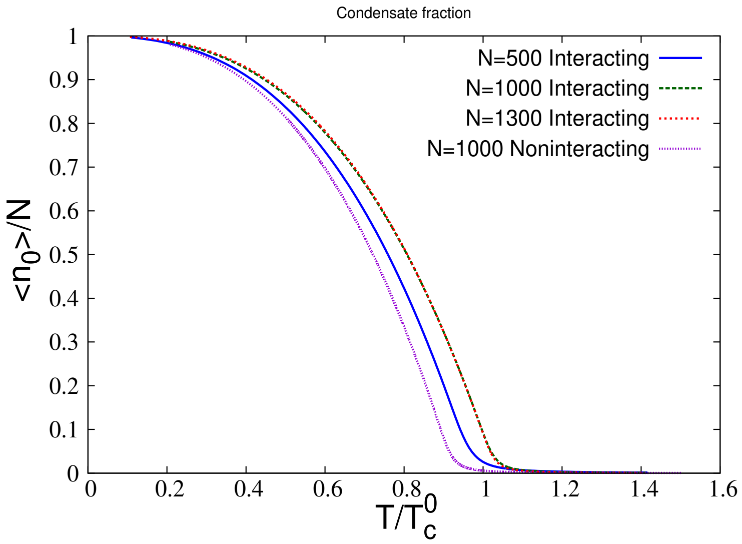

Figure 4.2: Condensate fraction ($< n_0 > /N$) is plotted with respect to reduced temperature $\frac{T}{T_c^0}$.

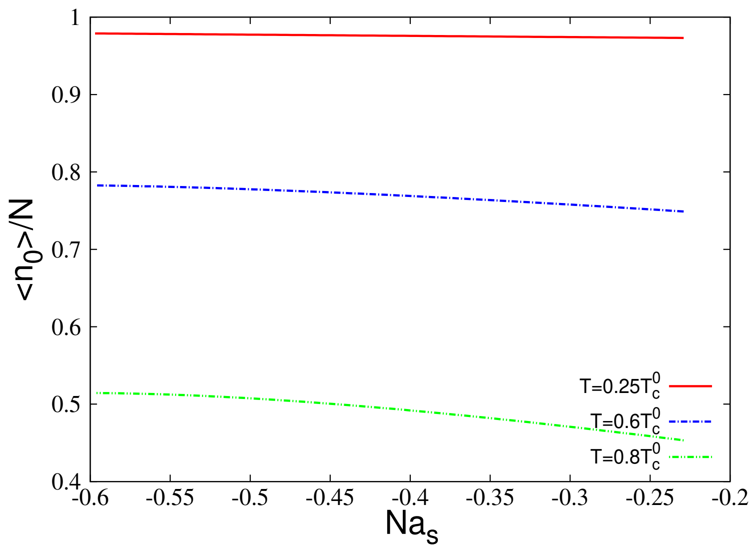

Figure 4.3: Condensate fraction ($< n_0 > /N$) is plotted with respect to the interaction strength (Na_s) for interacting atoms of ^7Li in the harmonic trap.

It is also important to discuss the effect of net interatomic interaction on the condensate fraction for a fixed temperature. In figure 4.3, we presented the condensate fraction for the interacting Bose gas with respect to the interaction strength parameter Na_s for the fixed set of temperatures $T = 0.2T_c^0$, $0.5T_c^0$ and $0.8T_c^0$. The condensate fraction curve for a specific temperature monotonically increases with respect to the non-interacting case. We know that for the increase in the attractive interaction strength parameter the local density of the condensate increases. So, the population in the ground state will also increase. The effect of interaction is clearly visible as condensate fraction increases with increase in interaction strength in comparison with the ideal Bose gas i.e., with no interaction ($Na_s = 0$). This basically supports the possibility of the enhancement in the condensation process with increase in the net interatomic interaction strength.

4.3 Fluctuation in the ground state atom number

Einstein taught us that the fluctuation in any system contains more information than the average or mean of any physical property. As fluctuation is more richer in demonstrating the state of a system, so we will discuss in detail about the fluctuation in the ground state atom number for attractive interacting BEC. The formula to calculate the root-mean-square fluctuation (δn_0) within the canonical ensemble approach has given in Chapter 3.3. In figure 4.4, the fluctuation in the condensate fraction (δn_0) is plotted against reduced temperature $\frac{T}{T_c^0}$. The fluctuation is zero at absolute zero temperature and then smoothly increases up to a specific temperature. At this specific temperature, known as the critical temperature, it sharply drops to zero. We know, from laser physics the fluctuations is more prominent near the threshold. However, for BEC, we see the threshold inversion. The accumulation in the number of photons for the lasing mode resembles the depletion in the number of atoms for condensate fluctuation. It can safely be said that the rate of loss of bosons from the ground state is equal to the rate of gain of the bosons in the ground state of the condensate. Thus, the fluctuation is almost zero at absolute zero temperature. However, at the critical temperature almost all the particles occupy higher excited states becomes normal Bose gas diminishing the population in the ground state. For uniform and homogeneous gas (trapless gas), the thermal fluctuation in condensate fraction may destabilize the system however it leads to the stable condensed state for the inhomogeneous mesoscopic system. To know more about the nature of the fluctuation, we study the temperature dependence of the fluctuation in the ground state atom number. We plot δn_0 versus $\frac{k_B T}{\hbar\omega}$ for several number of particles in figure 4.5. We see similar kind of behaviour of δn_0 as we have seen with the reduced temperature. Fluctuation dies out at absolute zero temperature and they are independent of particle number at very low temperature. The curves give maximum fluctuation around the transition point T_c^0 and above T_c^0 they sharply fall to zero showing no fluctuation in the ground state. All the curves for the finite size interacting bosons deviate

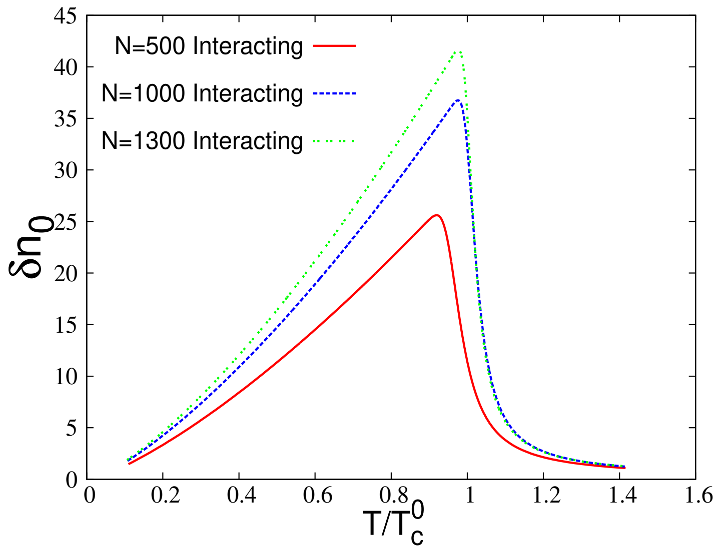

Figure 4.4: Fluctuation in the ground state atom number (δn_0) is plotted with respect to reduced temperature ($\frac{T}{T_c^0}$) for interacting and noninteracting atoms of ^7Li in the harmonic trap.

from the thermodynamic limiting curve which is plotted by utilizing the following formula :

$$\begin{aligned} < \delta^2 n_0 >= \Big(\frac{k_B T}{\hbar\omega}\Big)^3 \zeta(2) + \\ \Big(\frac{k_B T}{\hbar\omega}\Big)^2 \Big[\frac{3}{2} ln(\frac{k_B T}{\hbar\omega}) + \frac{3}{2}\gamma + \frac{5}{4} + \zeta(2)\Big] - \\ \frac{1}{2}\Big(\frac{k_B T}{\hbar\omega}\Big), \end{aligned} \tag{4.1}$$

where $\gamma = 0.57722$ is Euler's constant and $\zeta(2)$ is the Riemann Zeta functions. As the attractive interacting system is in mesoscopic regime, the atom number is

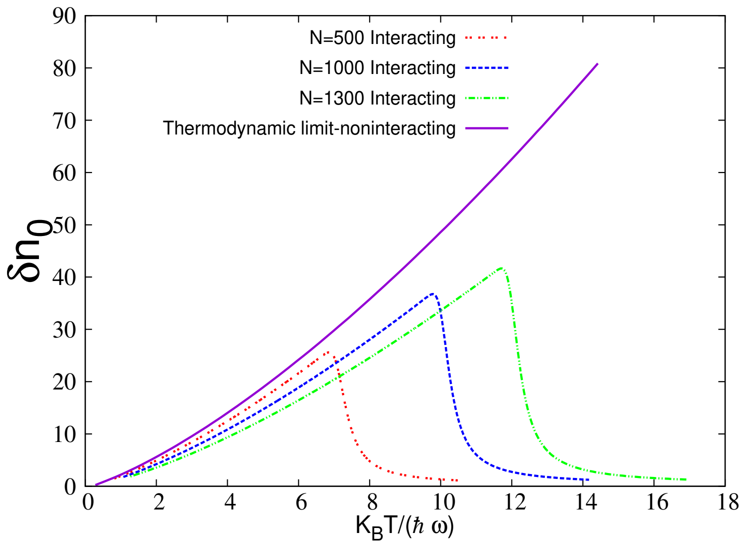

Figure 4.5: Fluctuation in the ground state atom number (δn_0) is plotted with respect to $\frac{k_B T}{\hbar\omega}$ for interacting and noninteracting atoms of ^7Li in the harmonic trap.

far from the thermodynamic limiting value. Due to the strong pair correlation for attractive interacting BEC finite-size effect is very prominent - deviation from the thermodynamic limit is visible. So, the fluctuation curve for the interacting case is away from the same for ideal thermodynamic limiting case. Thus, the plots nicely exhibit that at large particle limit the fluctuations are expected to have the desired behaviour. At much lower temperature, the fluctuations are independent of particle number and at the critical temperature they suddenly fall to zero. Unlike our previous observation for repulsive BEC [43], for attractive BEC we observe the dependence of N in the moderate temperature well below to achieve maxima. At higher temperature, all the graphs sharply fall to zero as expected. However, the sharp fall indicates the possibility of phase transition even in the mesoscopic regime.

We have also studied the effect of interaction strength on the fluctuation in the condensate fraction along with the temperature dependence. For a fixed temperature $T = 0.2T_c^0$, we have plotted δn_0 against the net interaction strength Na_s [see figure 4.6] and the curves show monotonically increasing nature with the increase in the net interaction strength. The curves moving from a lower to a higher value

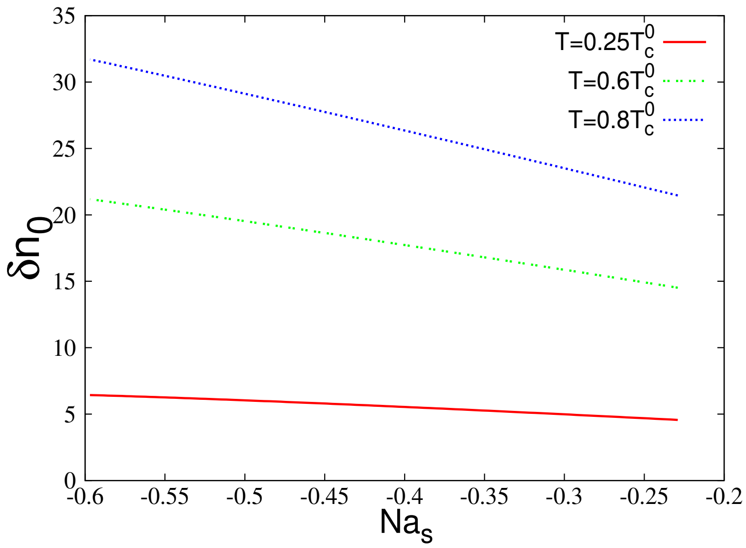

Figure 4.6: Plot of δn_0 with respect to interaction strength Na_s for interacting and noninteracting atoms of ^7Li in the harmonic trap.

of δn_0 show increase in the population of the ground state. We can say that, the attractive interaction enhances the condensation process for a fixed temperature. We have repeated the same for the other two temperatures $T = 0.5T_c^0$ and $T = 0.8T_c^0$ which also support the same conclusion as for the low temperature.

4.4 Central moments of the fluctuation

According to the analytical formula [Eq. 3.17], given in Chapter 3.3, we can calculate several orders of statistical moment (cumulants) of fluctuation in the ground state occupation. Here, we will present the result for second-, third- and fourth-order of central moment. Second-order central moment is usually termed as the root-mean-square fluctuation or standard deviation ($\sigma =< \Big(n_0 - < n_0 > \Big)^2 >$). All the moments are normalized with N. In figure 4.7, we have shown the variation of standard deviation for attractive interacting BEC for the following set of particles : N = 500, 1000 and 1300. For comparison, we have plotted the same for noninteracting Bose gas. Standard deviations for the interacting Bose gas are shifted towards right with respect to that for the noninteracting gas. So, the standard deviation is larger for the interacting case though the qualitative nature of all the curves are the same.

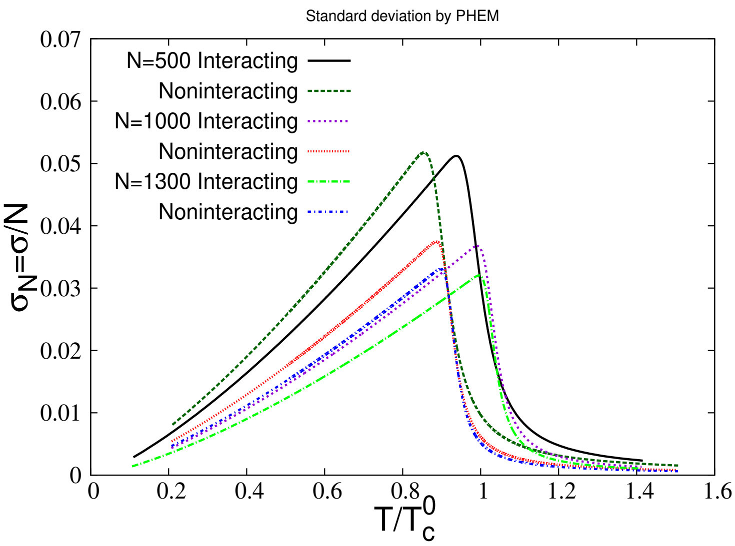

Figure 4.7: Standard deviation normalized with N is plotted against the reduced temperature T/T_c^0 for indicated numbers of interacting and noninteracting atoms of ^7Li in the harmonic confinement.

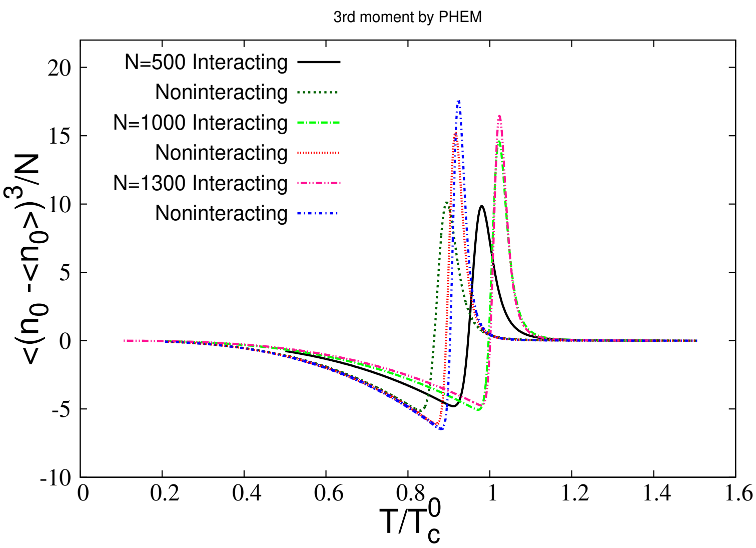

Figure 4.8: Third-order central moment normalized with N is plotted against the reduced temperature T/T_c^0 for indicated numbers of interacting and noninteracting atoms of ^7Li in the harmonic confinement.

All the curves smoothly increase up to the critical temperature and then drop to zero. So, the possibility of phase transition can be predicted and we can calculate the critical temperature T_c for the interacting BEC, where the peak of the moment

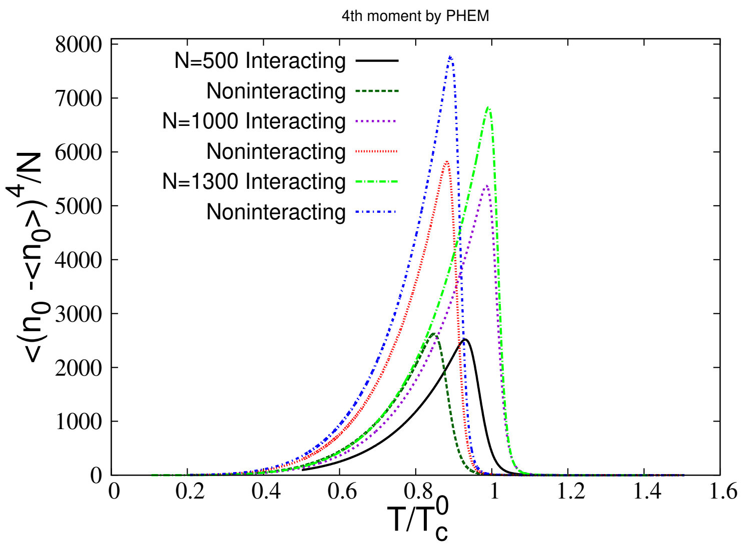

Figure 4.9: Fourth-order central moment normalized with N is plotted against the reduced temperature T/T_c^0 for indicated numbers of interacting and noninteracting atoms of ^7Li in the harmonic confinement.

arises.

In figure 4.8, we plot the variation of normalized third-order central moment of fluctuation $<\Big(n_0 - <n_0>\Big)^3>$ as a function of reduced temperature $\frac{T}{T_c^0}$ for both attractive interacting and noninteracting system with the same set of particle number as reported above. The curves show a sharp peak around T_c^0 but they are not Gaussian in nature. Fourth-order central moment for interacting ^7Li BEC is plotted as a function of reduced temperature $\frac{T}{T_c^0}$ in figure 4.9. The comparison is also made with the noninteracting case - they smoothly increase up to critical temperature showing more fluctuation for the attractively interacting BEC. The sharp fall of the curves across the T_c is as expected.