Chapter 9

Atom number fluctuation calculation through the collapse revival phenomena of correlation functions and Shannon information entropy

In the previous chapter, we discussed about the dynamics of one-body, two-body density and Shannon information entropy for three bosons in a triple well optical lattice interacting via contact interaction. Here, we will observe the time evolution of Glauber's normalized correlation functions. The effect of dipolar interaction on the correlation functions and on the entropy production are the main focus. The first- and second-order Glauber's normalized correlation function at many-body level is calculated which may provide additional tools to study the pathway from localization to delocalization. The normalized first-order correlation function is defined by

$$g^{(p)}(x'_1, ..., x'_p, x_1, ..., x_p; t) =$$

$$\frac{\rho^{(p)}(x_1, ..., x_p | x'_1, ..., x'_p; t)}{\sqrt{\prod_{i=1}^p \rho^{(1)}(x_i | x_i; t) \rho^{(1)}(x'_i | x'_i; t)}}.$$
(9.1)

It is the key quantity to define the spatial *p*-th order coherence [203, 204]. Here, $\rho^{(p)}(x_1, ..., x_p | x'_1, ..., x'_p; t)$ is the *p*-th order reduced density matrix of the state $|\Psi\rangle$. In the case of $|g^{(p)}(x'_1...,x'_p,x_1...,x_p;t)| > 1$ (< 1), the detection probabilities of p particles at positions $x_1, ..., x_p$ are referred to as (anti-)correlated. p = 1, 2 provide the first- and second-order Glauber's normalized correlation functions respectively. We will see in the result section how the loss of first-order coherence and emergence of anti-bunching effect in the second-order coherence is linked with the Shannon information entropy. The timescale of loss of coherence in first-order correlation function is the connecting link between the entropy production. Due to the long-range tail of the dipolar interaction, the time scale for phase transition from superfluid to Mott insulator differ a lot from the contact interaction.

We prepared the dipolar interacting system in the initial state with the lattice depth V = 3.0 and dipolar interaction strength $g_d = 0.01$ which is a SF phase exhibiting both inter- and intra-well coherence and tunneling is allowed. To quench the system, we instantaneously increase the lattice depth to V = 10.0 keeping g_d fixed. For comparison, we present the results for the quench dynamics with contact interaction. The time evolution of the normalized first-order Glauber's correlation function and Shannon information entropy are studied for both the dipolar and contact interaction. Due to change in the interaction strength and lattice depth, the atoms in the lattice fluctuate. The atom number fluctuation can be understood by studying the phase diffusion due to the quantum fluctuation. Atom number fluctuation is related with the collapse - revival of the phase coherence. The ratio of the two characteristic times - $t_{collapse}$ and $t_{revival}$ is the measure of atom number fluctuation for the few bosonic systems. Here, $t_{collapse}$ is the time when the system enters in the fragmented Mott phase and $t_{revival}$ is the time when the system revives the initial SF phase. The experimental observation of atom number fluctuation [83] for few-boson system can be verified from our theoretical calculation.

9.1 Natural occupation

We analyze the eigenvalues of the reduced one-body density matrix, the natural occupations [172], as a function of the interaction strength between the particles (dipolar and contact) [see figure 9.1]. For the lattice depth quench, in a dipolar

interacting system, the occupation of the first natural orbital is almost 100% at the initial time and all the other higher orbitals have no significant population [see 9.1(a)]. This is condensed superfluid (SF) phase. The population in the



Figure 9.1: (a) Natural-orbital occupations as a function of time for lattice-depth quench to V = 10.1 for dipolar interaction. (b) Natural-orbital occupations as a function of time for lattice-depth quench to V = 10.1 for contact interaction. Computations are done with M = 6 orbitals. The initial *SF* phase fragments with time and all three natural orbitals are occupied. In both the occupation plots shown above n_2 and n_3 overlap with each other due to translational invariance. The contribution from all the other higher orbitals are negligible and they almost overlap. All quantities are dimensionless.

first orbital decreases with increase in time while the other higher orbitals start to be populated. The system shows exactly three-fold fragmentation at time t = 21.0, when it enters the MI phase. Again, the first orbital starts to be populated and the population in the other higher orbitals start decreasing. We see oscillatory behaviour in natural orbital occupation and at time t = 58.0 we again infer three-fold fragmentation. For comparison, we plot the evolution of natural occupation for lattice depth quench with contact interaction in figure 9.1(b). The initial SF state becomes fragmented and the system enters the fragmented MIphase for the first time at t = 31.0 which is significantly larger than the time required for fragmentation for dipolar interaction [t = 21.0]. This significantly slower $SF \rightarrow MI$ transition i.e., a larger characteristic time to reach MI phase distinguishes the lattice depth quench for the dipolar and contact interaction. Due to the long-range tail of the dipolar interaction, the fragmentation process is faster.

9.2 First-order Glauber's correlation function

We now start to discuss the time evolution of the first-order correlation function for the dipole-dipole interaction. In figure 9.2, we present the time evolution of the normalized first-order Glauber's correlation function $|g^{(1)}(x'_1, x_1; t)|^2$ as a function of two spatial variables x'_1 and x_1 for various time t. Initially, at time t = 0, the correlation function remains close to unity for all (x'_1, x_1) , therefore the system is fully coherent. With increase in time, as fragmentation is built up in the many-body state, the off-diagonal $(x'_1 \neq x_1)$ correlation is gradually lost. At



Figure 9.2: Time evolution of the normalized first-order Glauber's correlation function $|g^{(1)}(x'_1, x_1; t)|^2$ for forward lattice depth quench from $V_i = 3.0$ to $V_f =$ 10.0 for dipolar interaction. We observe collapse $(SF \to MI)$ -revival $(MI \to SF)$ dynamics. See the text for details. All quantities are dimensionless.



Figure 9.3: Time evolution of the normalized first-order Glauber's correlation function $|g^{(1)}(x'_1, x_1; t)|^2$ for forward lattice depth quench from $V_i = 3.0$ to $V_f =$ 10.0 for contact interaction. We observe collapse-revival dynamics (see text). All quantities are dimensionless.

time t = 21.0, when the many-body state is completely fragmented, the correlation function is unity almost exclusively along the diagonal $(x'_1 = x_1)$ – this is called the collapse time $(t_{collapse})$. Away from the diagonal $(x'_1 \neq x_1)$, the correlation function is close to zero. The complete loss of the off-diagonal correlation characterizes the fragmented MI state. However, in long-time dynamics, the system again starts to build up coherence. At time t = 58.0, both inter- and intra-well coherence is regained and $|g^{(1)}(x'_1, x_1; t)|^2$ becomes unity for all (x'_1, x_1) - the system revives to its initial SF phase – this is called the revival time $(t_{revival})$.

We repeated the simulation for contact interaction to compare the observed dynamics and time scale with dipolar interaction. We observed that similar collapserevival dynamics is seen in the first-order coherence (figure 9.3). However, the time scale for contact interaction is much longer. For the same simulation with contact interaction, we observe $t_{collapse} = 31.0$ and $t_{revival} = 81.0$ as obtained from figure 9.3. We see for dipolar case $t_{collapse} = 21.0$ and $t_{revival} = 58.0$ where $t_{collapse}$ is the time for the system to enter in the fragmented MI phase and the $t_{revival}$ is the time when the initial full coherence is revived.

9.3 Shannon information entropy

The coherence and correlations are experimentally accessible, so we can make a connection between the entropy dynamics and the collapse-revival cycle observed in the correlation dynamics. For dipolar interaction, the Shannon information entropy, (calculated from Eq. 8.1), is plotted as a function of time in figure 9.4(a). The entropy slowly increases and reaches at a maximum value around t = 21.0. We observe a broad maxima from t = 21.0 to t = 54.0, which signifies that the system retains to its maximum entropy state which is a MI phase. $S^{info}(t)$ goes to minimum value at t = 58.0 - the same time when the many-body state revives to SF phase same as inferred from the first-order correlation function depicted in figure 9.2. Thus, the collapse-revival cycle in coherence dynamics basically exhibits the maximum-minimum entropy cycle in the evolution of entropy. We can define collapse time $t_{collapse}$ in the following three ways: (a) The system enters completely fragmented MI phase.

(b) The Shannon information entropy becomes maximum.

(c) The off-diagonal correlation is completely lost.

For the present simulation, the value of $t_{collapse} = 21.0$ is uniquely determined from the above three possible ways. Similarly, revival time $t_{revival}$ can be calculated in three ways: (a) The system revives to the initial SF Phase.

(b) Information entropy reaches its minimum value.

(c) The system becomes fully coherent. For the present simulation, $t_{revival} = 58.0$ is again uniquely determined by the above three criteria.

To compare the dynamics of Shannon information entropy for dipolar interaction, we redid the simulation for contact interaction. We observe similar collapserevival dynamics - the system passes from maximum to minimum entropy state



Figure 9.4: Time evolution of Shannon information entropy S(t) for forward lattice-depth quench $V_i = 3.0$ to $V_f = 10.0$: (a) dipolar interaction and (b) contact interaction. In both cases, entropy passes through maximum point (*MI* phase) and minimum point (*SF* phase) in their corresponding time scale. All quantities are dimensionless.

[figure 9.4(b)]. We calculated $t_{collapse}$ and $t_{revival}$ for the contact interaction from the above specified definitions as : $t_{collapse} = 31.0$ and $t_{revival} = 81.0$. Both timescales are larger than the corresponding dipolar interacting case. Due to the long-range repulsive tail, we found fast fragmentation process for dipolar interaction.

9.4 Second-order Glauber's correlation function

For dipolar interaction, the variation of normalized second-order Glauber's correlation function $g^{(2)}(x_1, x_2; t)$ as a function of x_1 and x_2 is presented in figure 9.5 for different time t. Initially, the system maintains second-order coherence for almost



Figure 9.5: Time evolution of the normalized second-order Glauber's correlation function $g^{(2)}(x_1, x_2; t)$ for forward lattice depth quench from $V_i = 3.0$ to $V_f =$ 10.0 for dipolar interaction. We observe collapse-revival dynamics (see text). All quantities are dimensionless.

all (x_1, x_2) . Over time, the diagonal coherence starts to fade out - the probability of detecting two particles along the diagonal decreases. Complete deletion of the diagonal is termed as anti-bunching effect - which appears at longer time (t = 35.0). However, at some longer times, the system regains second-order coherence. We observe the same collapse-revival phenomena as found in first-order coherence.

The anti-bunching effect is also developed in the second-order correlation function (figure 9.6) when we repeat the simulation for contact interaction. However, the time scale for contact interaction to lose the off-diagonal coherence and development of anti-bunching effect is longer than the dipolar interacting system as shown in figure 9.5.



Figure 9.6: Time evolution of the normalized second-order Glauber's correlation function $g^{(2)}(x_1, x_2; t)$ for forward lattice depth quench from $V_i = 3.0$ to $V_f =$ 10.0 for contact interaction. We observe collapse-revival dynamics (see text). All quantities are dimensionless.

9.5 Atom number fluctuation

The quantum collapse - revival dynamics, observed for lattice depth quench in 1D optical lattice show, how the atom number fluctuation varies during the quench process. The on-site atom number fluctuations can be probed by measuring the ratio of collapse time $t_{collapse}$ to revival time $t_{revival}$. In the present situation, with few number of bosons in the lattice, we characterize $\tau = \frac{t_{collapse}}{t_{revival}}$ as the figure of merit to reflect the width of the atom number distribution. Now we performed the lattice depth quench for several larger values of lattice depth, we observed that the collapse-revival dynamics scenario is maintained for both contact and dipolar interaction. Here, we will discuss about the effect of long-range tail of the dipolar



Figure 9.7: The ratio of collapse time $t_{collapse}$ to revival time $t_{revival}$ for different lattice depth quench:(**a**) for dipolar interaction where the yellow shading has the value 0.379 ± 0.026 , which is bit higher than the theoretical prediction of $0.128 \pm$ 0.002 for Poisson number distribution (Eq.11 of Ref. [83]); and (**b**) for contact interaction, where the purple shading has the width of 0.405 ± 0.03 , which is bit higher than the prediction given above. In both cases, τ is independent of lattice depth.

interaction on the atom number fluctuation. In figure 9.7(a), we plot the figure of merit τ as a function of lattice depth and we observe that τ is independent of the depth of the lattice potential. This τ is inversely proportional to the width of the distribution as prescribed in Ref. [83]. So, atom number fluctuation is also independent of the depth of the lattice potential. Similar to dipolar interaction, we calculate τ for contact interaction and presented as a function of lattice depth V in figure 9.7(b), which lies within the same band as depicted for dipolar interaction in figure 9.7(a). This observation is in agreement with previous experimental results (figure 2 of Ref. [83]) where the initial atom number distribution was considered as Poissonian and the width of the fluctuation was calculated from the collapse - revival phenomenon.